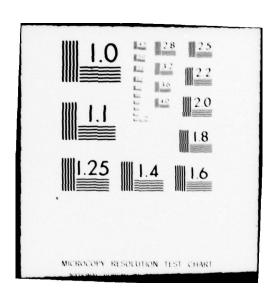
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20. Abstract continued.

of a nonlinear input/output map in terms of its Volterra transfer functions to a minimal realization in state bilinear form for discrete-time systems. The investigation of backward shift realizations for nonlinear systems has stimulated new thoughts on its use in linear systems theory. This has resulted in a comprehensive view of such topics as - minimal realization, minimal partial realizations, minimal realizations in the infinite dimensional case, and canonical forms.



The production of the

Interim Scientific Report

for

United States Air Force Grant No. AFOSR 77-3158

SYSTEM OPTIMIZATION BY PERIODIC CONTROL

Elmer G. Gilbert, Principal Investigator

Department of Aerospace Engineering

The University of Michigan

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Technical Information Officer

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Summary of Research Effort

This report summarizes research activities carried out during the period October 1, 1977 through September 30, 1978 under AFOSR Grant

No. 77-3158. The main concern was with the theory of periodic control and its applications. In addition, some questions concerning the realization of linear and nonlinear systems were studied. The attached chronological Bibliography includes: articles which appeared in journals during the year [1, 2], papers which appeared in conference proceedings [3, 4, 5], papers submitted for publication [6, 8], and a supplementary technical report [7]. The following paragraphs review these items along with additional activities which have not yet been put into papers.

The principal investigator was Elmer G. Gilbert. Daniel J. Lyons and Dennis S. Bernstein, doctoral students at the University of Michigan, also worked under the grant and made substantial contributions. A. E. Frazho, Assistant Professor of Electrical Engineering at the University of Rochester, worked with Gilbert on extensions of research they had begun in the previous year when Frazho was supported for several months under the grant. Support for Frazho's activities during the past year was limited to travel expenses (\$250) so he could come to Ann Arbor to work with Gilbert (two visits).

Items [1, 2] report the final publication of papers which were partly involved with last year's research effort. These papers were presented

at the 1977 Joint Automatic Control Conference in San Francisco. They received the O. H. Schuck Award of the American Automatic Control Council for being the best papers of the conference.

The article [5] is a revision of a paper which was prepared the year before. It was to be presented by Gilbert at the Seventh IFAC Congress in June at Helsinki. Unfortunately, a last minute family emergency prevented Gilbert from going. While IFAC Preprint Volumes are readily available, it would be better if the principal results appeared in a regular journal. In the coming year a new paper will be prepared to meet this need.

One of the objectives of the year's research was to set right an error which had been noted [a, b] in an important paper [c] having to do with periodic control. This paper considers second order conditions for optimality and it obtains a frequency domain test (the \$\pi\$ test) which can show that periodic operation may be better than optimal steady-state operation. The research in this direction proved successful and is described in [6, 7]. Item [6] gives the main results and has been submitted to the IEEE Transactions on Automatic Control. Item [7] contains supporting proofs which turned out to be quite lengthy. The principal results are: (1) a \$\pi\$ test for a somewhat more general problem than considered in [c], (2) a statement of a reasonable auxiliary condition which makes the \$\pi\$ test valid, (3) an exploration of the relationships between second order conditions for steady-state optimality and second order conditions for optimality in the dynamic problem. The approach to (3) is similar in spirit to the exhaustive

effort than was expected, but go beyond what was suggested in [b]. Moreover, the tools used in [6,7] are capable of generalization. Specifically,
it should be possible to extend the developments to include constraints
on the control (in a more general way than treated in [e]) and the second
order effects of strong variations in the controls. These directions will
be followed in the coming year and should yield stronger, more useful
frequency domain tests.

During the preceding year the application of periodic control to maximizing the specific range of an aircraft in cruise was considered. A simplified aircraft model was used and a specialized algorithm was developed to optimize the periodic motion (see [b, f] for additional detail). The preliminary results were interesting but numerical difficulties were encountered and truly realistic aircraft models could not be used. Since this year's effort on the cruise problem has been considerable, and a descriptive report has not yet been written, it will be described in some detail.

The difficulties and limitations of the previous work led to the investigation of new techniques for the numerical optimization of aircraft performance. The result is an optimization program which applies to the following rather general nondimensional aircraft model:

$$\frac{dV}{dx} = (V \cos \gamma)^{-1} (T - D - \sin \gamma)$$

$$\frac{d\gamma}{dx} = (V^2 \cos \gamma)^{-1} (L - \cos \gamma),$$

$$\frac{dh}{dx} = \tan \gamma,$$

where x = distance along earth's surface,

V = aircraft speed,

γ = flight path angle,

h = altitude,

D = D(L, V, h) = drag,

L = lift.

The controls are L and T. In the case of periodic control applied to the optimization of specific range (SR) the cost is

(2)
$$J = \frac{1}{x_1} \int_0^{x_1} \frac{\sigma(h, V) T}{V \cos \gamma} dx = (SR)^{-1}$$

where x_1 is the period and σ is the thrust specific fuel consumption. Periodic motion requires

(3)
$$V(0) = V(x_1), \quad \gamma(0) = \gamma(x_1), \quad h(0) = h(x_1).$$

In the algorithm, the altitude and speed are specified as spline functions and (1) is then used to compute γ , L, and T. This "backward approach" allows the periodicity conditions (3) to be imposed easily, which is not the case in conventional algorithms where L and T would be specified.

Also, (1) is solved directly without the complications of an additional integration algorithm. Constraints on the control and state variables are imposed by penalty functions. Since the splines are specified by their values at the joints, the optimization is over a finite number of parameters. The PRAXIS program which does not require derivative information has been used for the minimization. The overall optimization program is very flexible (e.g., the joints of the splines may be specified arbitrarily) and is written in modular form. Thus, it is easily adapted to other performance functions and constraints and to different aircraft data. Early experience with the algorithm has been very successful.

The principal application has been to a parametric study of the specific range problem started last year. In this problem it is assumed that: σ is constant, air density is exponential in h, L and D are related by a classical (subsonic) lift-drag polar, and $h \leq h_{max}$. The problem is fully described by two nondimensional parameters:

$$\delta = \frac{1}{2} \min \frac{D}{L},$$

$$\beta = \frac{v_s^2}{gh_s},$$

where V_s = the maximum endurance steady-state cruise speed at h = h_{max},
g = acceleration due to gravity,

h = scale height of exponential atmosphere.

The Figure summarizes the results of the parametric study. It required

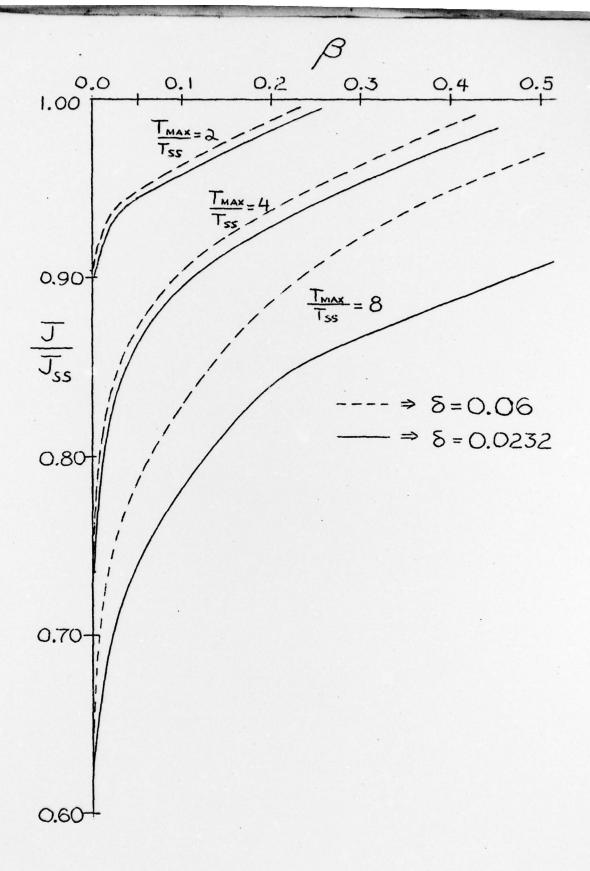


Figure. Cost improvement in aircraft cruise.

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30 separate optimizations of the problem (1), (2), (3). It was assumed that the thrust was constrained by $0 \le T \le T_{\text{max}}$ (see Figure) where $T_{\text{ss}} = 1$ thrust required for optimum steady-state cruise at 1 = 1 has. Since 1 = 1 is the optimum steady-state specific range, 1 = 1 implies periodic cruise is better than optimum steady-state cruise. Parameter values such as 1 = 1 such

The work begun last year on nonlinear systems and realization theory has been carried further. The results in [g] (see [f]) have been extended to cover impulsive kernals and discrete-time systems. These extensions will be incorporated in a larger version of [g] which will be submitted for publication. The nonlinear realization techniques studied by Frazho and Gilbert last year were refined considerably. The key tool is an abstract backward shift realization. Partial results are reported in [3, 8]. For discrete-time systems, they show how to go from the specification of a nonlinear input/output map in terms of its Volterra transfer functions to a minimal realization in state bilinear form. Furthermore, these ideas

extend to a wider class of nonlinear input/output maps, e.g., those which have state affine realizations. These extensions are being pursued by Frazho.

The investigation of backward shift realizations for nonlinear systems have stimulated new thoughts on its use in linear systems theory. This has resulted in a comprehensive view of such topics as: minimal realizations, minimal partial realizations, minimal realizations in the infinite dimensional case, canonical forms. The paper [4] gives some insight into these matters and emphasizes the partial realization problem.

Specifically, the restricted backward shift realization is used as a tool to give a simple, complete characterization of the family of minimal partial realizations. Gilbert and Frazho intend to write additional papers in this direction. Perhaps the most significant effect of this research on linear systems will be its implications with respect to nonlinear problems.

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